Formative Assessment in Mathematics

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I am a big fan of using definitions as a starting point for thinking about a topic, so let us look at a definition of *assessment* from the National Council of Teachers of Mathematics (1995): "Assessment is . . . the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (3).

Depending on your age, this definition may describe the experience you had with assessment in mathematics during your school career, but for most readers, "testing" was really the only kind of "assessment" we knew. Like clockwork, at the end of every few sections of our math book, there would be a quiz (for a GRADE) and at the end of every chapter, there would be a TEST (for a MAJOR GRADE). Then, no matter what grades any of us received, we would go off to the next chapter, where the cycle began again.

That type of testing (of which there are many varieties) is known in today's parlance as "summative assessment," defined as "a culminating assessment, which gives information on student's mastery of content" (Association for Supervision and Curriculum Development 1996, 60). Table 1 summarizes the principal characteristics of summative assessment.

In contrast, the focus of this special issue of *The Clearing House* is "formative assessment" (also see table 1). For the purposes of this article, the following definition of formative assessment will be used: "assessment which provides feedback to the teacher for the purpose of improving instruction" (ASCD 1996, 59). This concept of assessment meshes nicely with the NCTM definition shown above.

Formative assessment—with or without that name—has always been found in the classroom, to an extent that depended on individual teachers' attitudes toward assessment. The teacher who believes, as Grant Wiggins does, that "good teaching is inseparable from good assessing," already uses an ongoing cycle of teaching, assessment, assessment of the teaching, reteaching (as necessary), assessment, teaching, and so on. "Assessment should serve as the essential link among curriculum, teaching, and learning" (Wilcox and Zielinkski 1997, 223).

Although there can be overlap between some types of formative and summative assessments, and although there are both informal and formal means to assess students, in this article, I will primarily offer suggestions for informal formative assessment for the mathematics classroom. The ideas presented will fall into three categories (adapted from Clarke and Wilson 1996):

TABLE 1
Characteristics of Summative and Formative Assessment

	Summative assessment	Formative assesssment
Time	At the conclusion of a learning activity	During a learning activity
Goal	To make a decision	To improve learning
Feedback	Final judgment	Return to material
Frame of reference	Sometimes normative (comparing each student against all the others); sometimes criterion (evaluating each student according to the same criteria)	Always criterion (evaluating all students according to the same criteria)

Source: Adapted from R. Pregent. 1994. *Charting Your Course: How to Prepare to Teach More Efficiently.* Madison, Wis.: Magna Publications, Inc.

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FIGURE 1
Assessing Mathematical Content Knowledge

Indicate your knowledge of each word/phrase below by writing a 1, 2, 3, 4, or 5 in front of it. The numbers signify the following five statements.

1. I've never seen the word/phrase.
2. I've seen the word/phrase, but I don't know what it means.
3. I know the word/phrase has something to do with . . .
4. I *think* I know what it means in math.
5. I know the word/phrase in one or several of its meanings, including the meaning for mathematics.

Using Measures and Equations

continuous	supplementary angles
opposites	acute triangle
line	equation
length of a segment	equivalent equations
ray	irrational number
central angle of a circle	perfect cube
complementary angles	absolute value
vertical angles	segment
right triangle	congruent segments
solving an equation	vertex of an angle
rational number	straight angle
perfect square	congruent angles
discrete	obtuse triangle
scientific notation	solution
endpoint	square root
midpoint	real number
angle	cube root
right angle	

- The student's mathematical *content* knowledge
- The student's mathematical *processes*, such as reasoning, communicating, problem solving, and making connections
- The student's mathematical *disposition*, such as attitudes, persistence, confidence, and cooperative skills

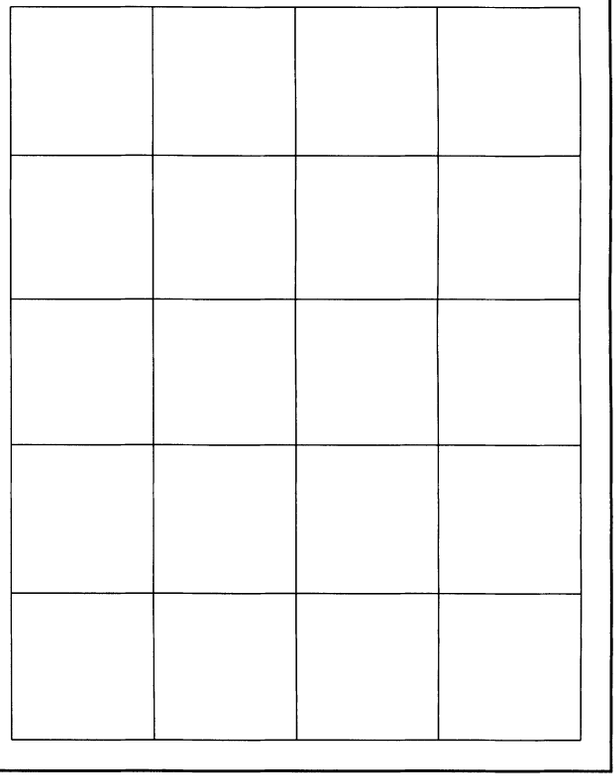
Assessing Mathematical Content Knowledge

If you agree with the notion that words are labels for concepts, then you will want to use the ideas shown in figure 1. I use this exercise as an informal pre- and post-assessment. At the beginning of a new unit or chapter (and again at the end), I give students a sheet similar to this one, with vocabulary terms for the unit listed. The first time you use the exercise, it is necessary to go over the five different levels of work knowledge, but students easily understand the idea that there are words they have never heard of and words that they know in several ways—and everything in between. (It is important to read the words aloud to the class because there are some words that students recognize when they hear them but not when they see them.) Then, have the students write their best understanding of all the words they rated as 4s or 5s so you can assess content knowledge. This exercise is not used for a grade but rather as formative assessment to give the teacher an idea of students' understandings of the concepts before and after the unit of instruction.

A second way of assessing students' content knowledge is the daily quiz sheet (Columba and Dolgos 1993). Each student receives a sheet (as shown in figure 2) at the beginning of the week. Then, each day, either as students enter class or as the closing activity for the day, four problems from a previous day's lesson or homework are given, and students enter each problem (and solution) in the four spaces for the day. The teacher can check these quickly or have a row grader check them. The sheets may be collected each day or at the end of the week, depending on the teacher's plan for using the assessment information.

The third suggestion for formative assessment of content knowledge is performance assessment. Entire books have been written on that subject; even though I cannot fully explain it in the context of this article, I would be remiss not to mention it. Performance assessments are assessments "in which students demonstrate in a variety of ways their understanding of a topic or topics. These assessments are judged on predetermined criteria" (ASCD 1996, 59). Baron (1990a, 1990b, and 1991) in Marzano and Kendall (1996) identified a number of characteristics of performance tasks. Such tasks

FIGURE 2
Daily Quiz Sheet



- are grounded in real-world contexts,
- involve sustained work and often take several days of combined in-class and out-of-class time,
- deal with big ideas and major concepts within a discipline,
- present non-routine, open-ended, and loosely structured problems that require students both to define the problem and to construct a strategy for solving it,
- require students to determine what data are needed, collect the data, report and portray them, and analyze them to discuss sources of error, and
- necessitate that students use a variety of skills for acquiring information and for communicating their strategies, data, and conclusions. (93)

Figure 3 is an example of a performance task from a book that is must reading for anyone interested in performance assessment, *A Teacher's Guide to Performance-Based Learning and Assessment* (Educators 1996), written by teachers from a Connecticut school district. I encourage all readers of this issue to learn as much as they can about performance assessment. It is powerful—and it will change your teaching forever!

FIGURE 3
Sample Math Performance Task: Fast Food Math

Background
A large national hamburger fast food restaurant chain printed an advertisement that read, "Seven percent of all Americans eat in our restaurants each day." This chain has 9,167 restaurants. There are approximately 250,000,000 American citizens.

Task
You are an employee of a consumer advocate group studying truth in advertising. You have been asked to consider this advertisement. Do you think it is probably true or probably false? Your task is to use logical mathematical procedures to analyze the claim this advertisement made and write a summary of your opinion.

Audience
The audience for your study and written summary is the person in charge of the consumer advocate group.

Purpose
The purpose of your analysis and written summary is to show the use of logical mathematical procedures to examine claims made in advertising.

Procedure
1. Review the assessment list for fast food math.
2. Rewrite the task in your own words.
3. Make a list of all the information that is clearly given.
4. Make a list of what needs to be estimated, assumed, or found out.
5. Make those estimates and/or assumptions.
6. Get other information that you need.
7. Use your math skills to analyze the advertisement.
8. Write a summary of your findings for your boss at the consumer advocacy group.

Source: Educators in Pomperaug Regional School District #15. 1996. *A Teacher's Guide to Performance-Based Learning and Assessment.* Middlebury, Conn.: Pomperaug Regional School District #15. Copyright 1996. Reprinted

Assessing Mathematical Processes

Many of us would love to open up our students' heads so we could see the wheels turning as they solve problems. Because that is not possible, we have to find ways to get the students to show us what they are doing. With some students, that is fairly simple because they are extremely metacognitively aware and don't mind letting others in on their secrets. Other students, though, rarely "think about their thinking" and so, of course, rarely let other people in on how they process (because they aren't even aware of it themselves). The following ideas work with both groups of students (and everyone in between); we just have to be more patient with the latter group. The good news is that once we begin asking adolescents to reflect on their thinking and problem solving, it starts happening.

Interviewing students is a powerful way to tap into their processing. In the best of situations, teachers would have time to interview students on an individual basis—at length. In reality, we don't have the time to do this, so we need to make time to (*a*) briefly interview students on an individual basis and/or (*b*) interview groups of students—either briefly or at length. Long and Ben-Hur (1996) found that the following phrases or statements are helpful when interviewing students who may not be fully forthcoming or expansive in their answers:

- I am interested in your thinking.
- Please help me understand. Suppose you are the teacher and I am your pupil.
- I don't think that this problem is easy. Sometimes I get confused . . . don't you?
- Sometimes when I have difficulties with a problem, I break it down into small steps. Let's do that here and find out . . . . [The problem is modified.]
- I like it when you take the time to think.
- I understand now, but . . . . (107)

Another marvelous idea for assessing students' mathematical processes is shown in figure 4 (adapted from Greenwood 1996). By using this evaluation sheet, we give students an opportunity to think about their processing—and to give examples of what it is that they do. The first time I use the sheet, I model what it is I want students to do and the specificity of examples I am looking for. Then, I accept nothing less than what I *know* that they can do. Students are sometimes resistant to this approach because it involves a kind of thinking that some are not used to, but I persevere, and within a short period of time am able to get the kind of formative assessment data that is helpful to me in my teaching.

Regardless of whether we are teaching seven-year-olds, seventh graders, or seventeen-year-olds, part of our task is

FIGURE 4
Student/Teacher Mathematics Evaluation Sheet

What I'd give myself	What the teacher would give me	Use the following code to mark yourself in each criterion. A = Always M = Most of the time O = Occasionally S = Seldom N = Not at all	Example(s)
		1. I can give clear and understandable explanations of the thought processes I go through when I am solving a problem.	
		2. I can use the materials to show that the mathematics I do makes sense to me.	
		3. Whenever I get stuck on a problem, I can use what I know to get unstuck.	
		4. I am able to identify errors in answers, in the use of mathematics materials, and in my thinking.	
		5. When I do a computation, I don't always need paper and pencil.	
		6. When a strategy doesn't work, I try another one instead of giving up.	
		7. I can extend, or change, a problem by asking extra questions or posing different conditions.	
		8. I study and practice before tests and quizzes.	

to teach them our expectations. If we expect students to reflect on their thinking and problem-solving processes, then we must teach them to do so. We need to (*a*) model our own reflections, (*b*) ask them about their reflections, and (*c*) give them opportunities to reflect on their processes. In other words, we can't give up just because they are resistant to giving us information about their mathematical processing.

Assessing Mathematical Dispositions

Disposition is defined as "one's usual mood; temperament, a habitual inclination, tendency." With teenagers, it is not always easy to determine their disposition or temperament regarding anything, including mathematics. Too often, if their reference group has decided that "school is not cool," then it is mandatory that they use all their body language and facial expressions (and sometimes words) to indicate their disdain for our beloved subject. We mustn't accept this judgment at face value (no pun intended). The ideas in this section will allow you to determine your students' mathematical dispositions (sometimes without their knowing it!).

The first idea I want to offer came about almost by a fluke. Another teacher and I were creating an assessment instrument for some research we were conducting. We had about half a page left on the eight-page instrument and didn't want to waste the paper, so we decided to pose the question in figure 5 (McIntosh and Draper, in preparation). It turned out to be the best question of the whole assessment!

FIGURE 5
Student Self-Rating Scale

20. As a math student in this class, I rate myself on the following scale (put an X on the scale where you rate yourself).

<-----1----2----3-----4----5-----6----7-----8----9----10----->

Probably the worst in the class.	Not too bad, not too good.	Totally awesome! Maybe the best in the school.

I rated myself this way because

_____
_____
_____
_____
_____
_____
_____

Consider using this question at the beginning of the year, and then several other times throughout the school year, to get a sense of the changes taking place.

I have found that adolescent students are willing and able to be more truthful when asked to write their thoughts down than when asked to share them publicly. For this reason, I use learning logs as often as possible to learn about students' dispositions toward mathematics. The term *learning log* is not one that I originated, but it is one that fits my philosophy of how writing looks in the mathematics classroom. The first part of the term states the purpose of the writing: learning. The second part connotes a particular format, that is, running commentary. A log is not meant to be a polished piece of writing, taken through draft after draft. Commander and Smith (1996) defined the purpose of learning logs as "reflections on specific cognitive aspects of learning . . . [emphasizing] the connection or personal engagement with academic skills and techniques" (447).

In the book *Write Starts: 101 Writing Prompts for Math* (McIntosh and Draper 1997), we offered a variety of ways to use learning logs to assess students' attitudes, beliefs, and stereotypes about mathematics. The following are some of the writing prompts suggested:

- What does a mathematician look like?
- My ability to do math is . . .
- When I am in math class, I feel . . .
- Mathematics has good points and bad points. Here's what I mean . . .
- I study, I pay attention, I take notes, I read my math book, but I still don't get math. True or false? Explain your answer.

Students' answers to questions such as these provide insight to the teacher as she or he plans instruction. Ignoring students' dispositions toward mathematics is done at teachers'—and students'—peril.

Conclusion

We should not think that formative assessment is something that is added on to our already full curriculum. Formative assessment is part of good teaching. There should be a seamlessness between instruction and assessment. The word *formative* comes from the Latin word meaning "shape or form." Formative assessment has as its purpose to shape upcoming instruction. If you use or modify the ideas offered in this article, you will find that your instruction is more targeted and more effective. Then, you can design more ideas of your own—and share them with as many other teachers as possible.

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